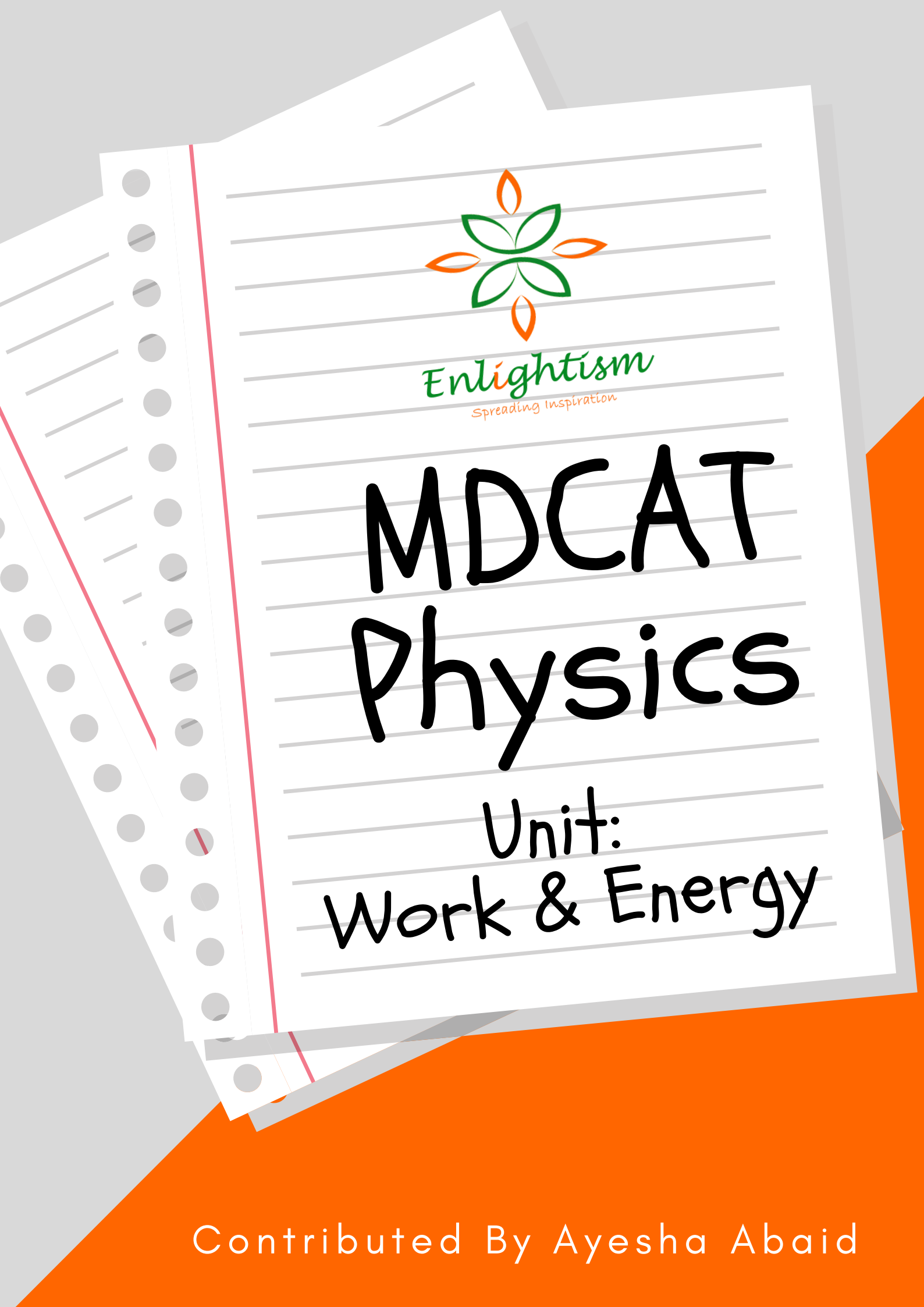
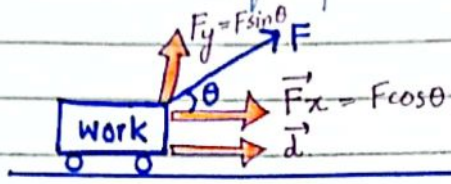

Enlightism
Spreading Inspiration
MDCAT
Physics
Unit:
Work & Energy
Contributed By Ayesha Abaid

WORK AND ENERGY

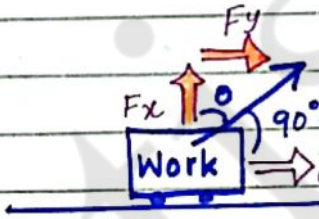
Work is done, if body is displaced by force in the direction of force.



(Dot/Scalar product of F & d)

Direction of $\vec{d}$ is in the direction of $\vec{F}$.

$$W = (F\cos\theta)d$$

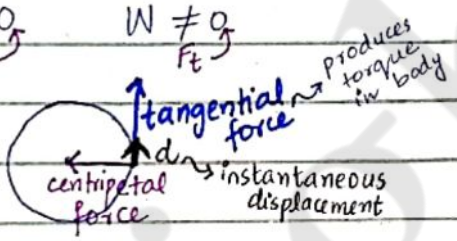


$W = (F\sin\theta)d = (F\cos(90-\theta)d$

Here $\vec{d}$ is in the direction of $F\sin\theta$, so it is not necessary, that $F\cos\theta$ will perform work. It totally depends upon nature of force (i-e component of force & θ)

Circular motion

$W = 0$ (F_c) $\quad W \neq 0$ (F_t)



→ non-uniform motion

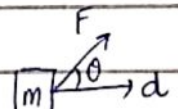
→ C.F ⊥ T.F

→ θ b/n F_t & d = 0° ($\because \cos 0° = 1$) so, $W \neq 0$

→ θ b/n F_c & d = 90°

→ $W = \tau\theta$

Mathematical Relations:-



(i) $W = \vec{F}.\vec{d} = Fd\cos\theta$

(ii) $W = P\Delta V = P(V_2 - V_1)$ (in gases)

($\Delta P \Delta V$) → isochoric procen, ΔV = const#,

(iii) $W = \tau\theta$ → circular motion

$W = rF_t\theta$ $\quad \theta = 90°$ b/n r & F

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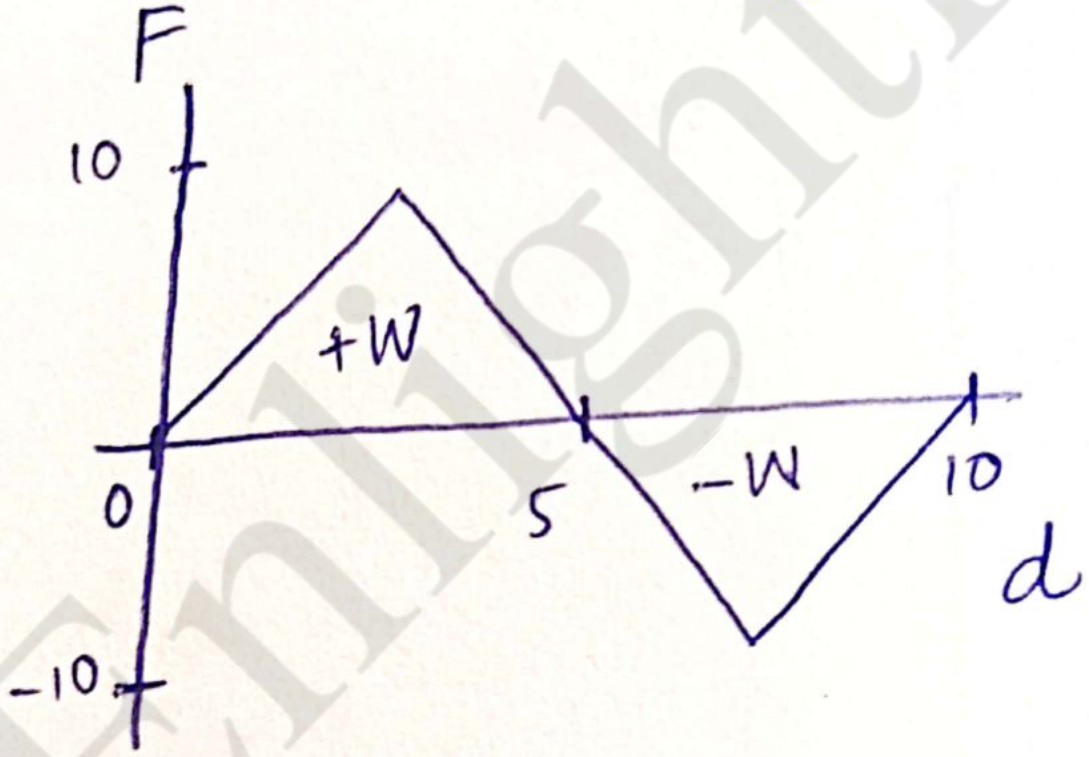
will perform work ... totally depends upon nature of force (i-e component of force e ...

Note:-



$W_{net} = 0$

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(iv) $W = VQ$ → when charge & Δ voltage is given (Potential difference)
$= \Delta V Q$ (not applied to capacitor)

(v) $W = \frac{1}{2} QV$ → For capacitor → ∵ average potential used

(vi) $W = \frac{1}{2} F x$ →

(viii) $W = \frac{1}{2} F \Delta l$ → Strain energy

Units:-

J ← SI unit → Nm

SI base unit kgm^2s^{-2}

$J = Nm$

Q $\vec{F} = 3\hat{i} + 2\hat{j} + 4\hat{k}\ N$

$\vec{d} = 2\hat{i} - 3\hat{j} + 5\hat{k}\ m$

$W = ?$

$W = \vec{F} \cdot \vec{d}$

$= (3 \times 2) + (2 \times -3) + (4 \times 5)$

$= 6 - 6 + 20$

$= 20\ J$

$A = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$B = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$\vec{A} \cdot \vec{B} = a_1a_2 + b_1b_2 + c_1c_2$

Q A body of mass 5kg, initially at rest, is moved by horizontal force 2N, calculate workdone by force in 10s.

$m = 5kg$, $F = 2N$, $V_i = 0$, $t = 10s$, $W = ?$

$d = V_it + \frac{1}{2}at^2$

$\Rightarrow a = \frac{F}{m} = \frac{2}{5}$ ∵ (not g)

$d = (0)(10) + \frac{1}{2}(\frac{2}{5})(10)^2$

$d = \frac{100}{5} = 20\ m$

$W = Fd$

$= 2 \times 20 = 40J$

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Types of Work (on basis of θ)

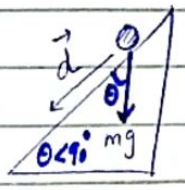
Positive work

$\vec{F}$ $\vec{d}$ θ

$\vec{F}$ along direction of motion

$0° \leq \theta < 90°$

(Acute angle)



Negative Work

$\vec{F}$ θ $\vec{d}$

retarding force (friction)

$\vec{F}$ opposite to direction of motion

$90° < \theta \leq 180°$

(obtuse angle)

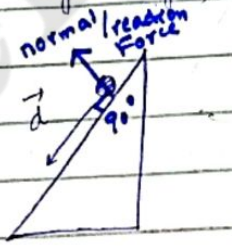
Zero Work

$\vec{F}$ ⊥ motion

$\vec{F}$ normal / ⊥ to direction of motion.

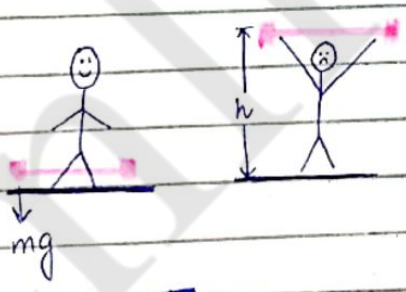
$\theta = 90°$

(right angle)



NOTE :-

Reaction force is always ⊥ to surface

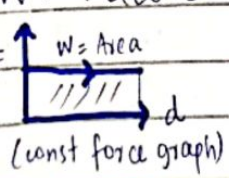


$W_{hold} = 0 \quad \because d = 0$

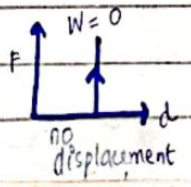
$W_{lifting} = \Delta P.E = mgH$

TYPES OF WORK (on basis of F magnitude)

Work done by const^t force

- $W = Fd\cos\theta$
- 

(const force graph)

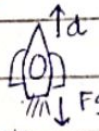


no displacement

Work done by variable force

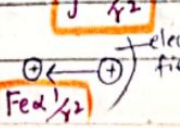
Example

- Work done by gravitational force



$Fg \propto \frac{1}{r^2}$

- Work done by electrostatic force



$Fe \propto \frac{1}{r^2}$

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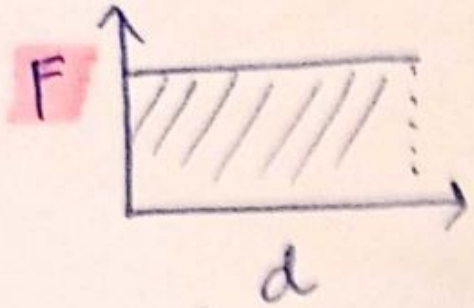
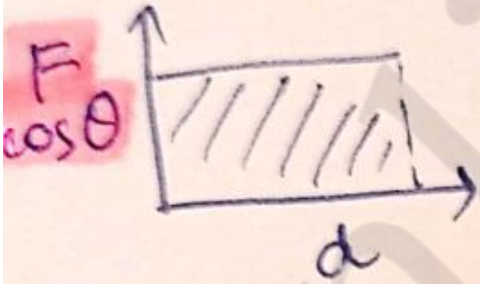
Graphical Representation of Work

if $\vec{F}$ along $\vec{d}$

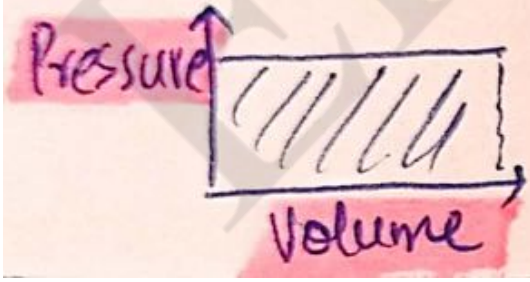


Area = work

if $\vec{F}$ at θ with $\vec{d}$



Area = work



Area = work

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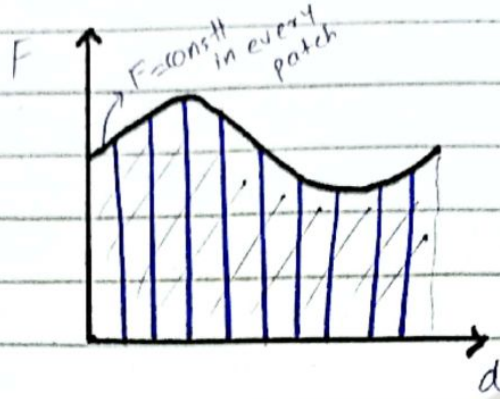
• Workdone by elastic field (Variable force)

$\uparrow F \propto x \uparrow$



Area = W

$$W = \sum_{i=1}^{n} F_i d_i \cos\theta_i$$

Note :-

Magnetic Force → Deflecting Force ← Centripetal force

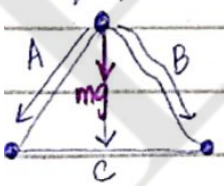
(Magnetic Force ⊥ to Magnetic field)

So, W = 0 (body attain circular path)

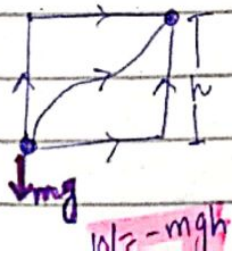
So, it is not conservative.

Conservative Force → P.E/T.E conserve

- Workdone is independent of path
- Workdone in closed path = 0



W = mgh



W = -mgh

Example :-

- Gravitational force
- Electric force
- Elastic force

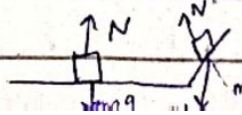
(Natural forces)

Non-conservative Force

- Workdone is dependent of path
- Workdone in closed path ≠ 0

Example :-

- Rocket propulsion force
- Motor propulsion force
- Frictional force
- Tension
- Drag force / Air resistance
- Normal force



So, W change

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$W = Fd\cos\theta$

$\theta = 0° \rightarrow W_{max} = +Fd$

$\theta = 30° \rightarrow W = \frac{\sqrt{3}}{2} Fd = 0.866\,Fd = 86.6\%\,F_{max}$

$\theta = 45° \rightarrow W = \frac{1}{\sqrt{2}} Fd = 0.707\,F_{max} = 70\%\,F_{max}$

$\theta = 60° \rightarrow W = \frac{1}{2} Fd = \frac{1}{2} F_{max} = 50\%\,F_{max}$

$\theta = 90° \rightarrow W_{min} = 0$

$\theta = 180° \rightarrow W_{max} = -Fd$

$\theta = 270° \rightarrow W = 0$

$\theta = 360° \rightarrow W_{max} = +Fd$

Torque ⟶ cross product → $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{r} = \hat{i} - 2\hat{j}$ m $\qquad \vec{\tau} = ?$

$\vec{F} = 3\hat{i} + 4\hat{j} - k$ m

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 0 \\ 3 & 4 & -1 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -2 & 0 \\ 4 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix}$$

$$= \hat{i}(2-0) - \hat{j}(-1-0) + \hat{k}(4+6)$$

$$= 2\hat{i} + \hat{j} + 10\hat{k}\ \text{Nm}$$

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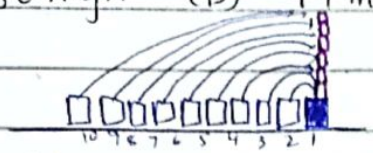
Q) 10 bricks each of thickness equal to 'h' and mass 'm' lie on table, how much work is required to stack them on the top of one another?

(A) 50mgh (B) 49mgh (C) 25mgh (D) none of these



$W = W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 + W_8 + W_9 + W_{10}$

$W = 0 + mgh + mg(2h) + mg(3h) + mg(4h) + mg(5h) + m(6h) + m(7h) + m(8h) + m(9h)$

$W = mg(h + 2h + 3h + 4h + 5h + 6h + 7h + 8h + 9h)$

$W = 45mgh$

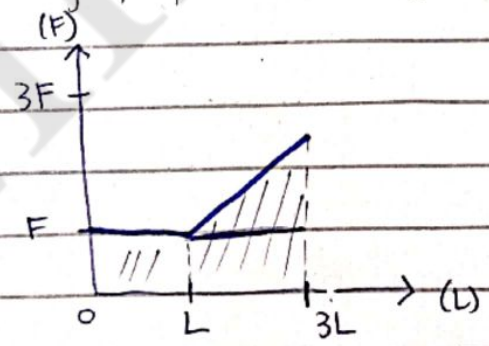
Shortcut:- Workdone on stacking 'n' bricks on one another

$$W_n = \frac{n(n-1)mgh}{2}$$

$n \rightarrow$ no. of bricks

$h \rightarrow$ height of each brick

Q A force 'F' acts on a body upto distance 'L' force when increases to '3F' while covering further distance '2L' Draw F-d graph / work diagram.



$W = (F \times L) + (F \times 2L) + (2F \times 2L)\frac{1}{2}$

$= FL + 2FL + 2FL$

$= 5FL$

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ENERGY

(ability to do work)

Two types but Many forms

Potential Energy

- w.r.t position
- could do work
- stored energy

Forms:

Chemical
gravitational
Nuclear
Elastic

Formulas:

$P.E_g = mgh$

$P.E_{absolute} = -\frac{GMm}{R}$

$P.E_{Elastic} = \frac{1}{2}kx^2$

$P.E_{Strain} = \frac{1}{2}F\Delta L$

$P.E_{electrical} = \frac{kQq}{r} = \Delta Vq$

Unit

SI → J

J = Nm

SI base → kgm^2s^{-2}

Other → eV, Btu (British thermal unit), KWh, calorie, ft·lb, erg

Smallest → eV

Biggest → KWh

$1\,eV = 1.6 \times 10^{-19}\,J$

$1\,Btu = 1055\,J$

$1\,KWH = 3.6\,MJ$

$1\,cal = 4.18\,J$

$1\,J = 0.7376\,ft\cdot lb$

$1\,erg = 10^{-7}\,J$

$1\,ft\cdot lb = 1.35\,J$

KWh > cal > J > erg > eV

Kinetic Energy

- w.r.t motion
- that does work
- motion / movement

Forms:

Electrical
Heat
Light, Sound

Formulas:

$K.E_{Trans} = \frac{1}{2}mv^2$

$K.E_T = \frac{P^2}{2m} = \frac{(mv)(mv)}{2m} = \frac{PV}{2}$

$K.E_T = \frac{1}{2}PV$

$K.E_{rot} = \frac{1}{2}I\omega^2$

$K.E = \Delta Vq$

→ $P.E = 1J$ $\quad P.E = mgh$ → if m is raised to 'h' ⇒ $\frac{1}{mg}$

$P.E = mg\left(\frac{1}{mg}\right) = 1J$

→ $K.E = 1J$ $\quad K.E = \frac{1}{2}mv^2$ → if m is moving with ⇒ $v = \sqrt{\frac{2}{m}}$

$K.E = \frac{1}{2}m\left(\sqrt{\frac{2}{m}}\right)^2 = 1J$

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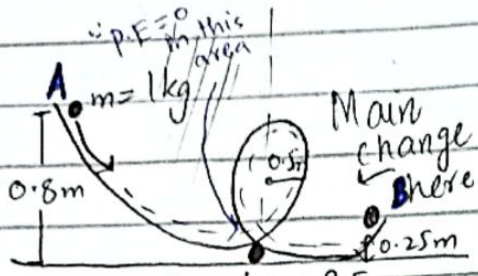
Note :- K·E is an example of **scalar / dot** product.

$$K.E = \frac{1}{2} m(\vec{v} \cdot \vec{v})$$

$$K.E = \frac{1}{2} mv^2$$

Q



$\Delta P.E_{AB} = ?$

$$P.E = mg\Delta h$$
$$= 1 \times 10(0.25 - 0)$$
$$= 10 \times \frac{1}{4} = 2.5\,J$$

When body comes from height 'h' to ground i-e P·E=0, then this area is not considered, P·E in next path is taken.

WORK ENERGY PRINCIPLE :-

→ Work is a form of Energy. ↓ Derived from 3rd eq. of motion

$W = \Delta E$

$W = (ma)d = \Delta K.E$

$W = \Delta K.E$

$Fd = \frac{1}{2} m(v_f^2 - v_i^2)$

$W = \Delta P.E$

$Fd = mg(h_2 - h_1)$

$W = \Delta P.E + \Delta K.E$

* $d \propto v^2$ → Q $v = 20m/s \xrightarrow{d=30m}$ break → rest ∵ $d \propto v^2$

same car $v' = 40m/s$ → break → rest $d' = ?$

i-e $v' = 2v \rightarrow d' = ?$

$$\frac{d'}{d} = \left(\frac{v'}{v}\right)^2 = \left(\frac{v'}{v}\right)^2 d$$

$$= \left(\frac{40}{20}\right)^2 (30) = 4 \times 30 = 120m$$

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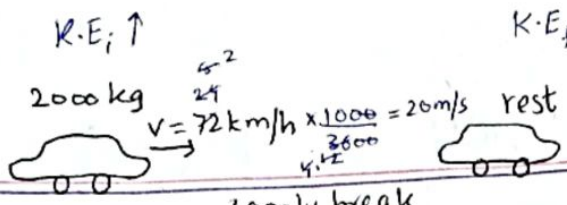

Q Magnitude of Retarding force = F = ?

Work Energy Principle

$\therefore W = Fd = \frac{1}{2} m(V_f^2 - V_i^2)$

$\Rightarrow F = \frac{1}{2d} m(V_f^2 - V_i^2)$

$F = \frac{1}{2(5)} (2000)(0 - (20)^2)$

$\Rightarrow = \frac{1}{10} (2000)(-400)$

$= -80000\,N$

$Mag = F = 8 \times 10^4\,N.$

Alternate Method:-

eq. of Motion → $2aS = V_f^2 - V_i^2$

$a = \frac{V_f^2 - V_i^2}{2S} = \frac{(0)^2 - (20)^2}{2(5)} = \frac{-400}{10} = -40\,m/s^2$

$\therefore F = ma$

$F = 2000 \times -40$

$F = -80,000\,N$

$Mag \rightarrow F = 8 \times 10^4\,N.$

Note:- imp

+W → P.E ↓

−W → P.E ↑

Conservative force →
- P.E is only associated with conservative forces.
- $K.E_i = K.E_f$

Non-conservative force →
- ~~P.E~~, No P.E
- $K.E_i \neq K.E_f$

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Q At which angle, work done will be 70% of its max value?

a) 30° **b) 45°** c) 60° d) 75°

$$W = W_{max} \cos\theta$$

$$70\%.\ W_{max} = W_{max} \cos\theta$$

$$\frac{70}{100} = \cos\theta$$

$$0.7 = \cos\theta$$

$$\theta = \cos^{-1} 0.7 = 45^\circ$$

Enlightsin

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INTERCONVERSION OF ENERGY

Case - I

(Neglecting Friction) i-e $F = 0$

→ $P.E_{loss} = K.E_{gain}$

or → $P.E_{gain} = K.E_{loss}$

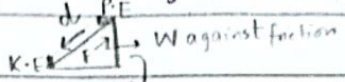
→ 100% conversion

$mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$

Case - II

(Do not neglect friction) i-e $F \neq 0$



① $P.E_{loss} = K.E_{gain} + \text{Work done against friction}$

$mgh = \frac{1}{2}mv^2 + Fd$

$v = \sqrt{\frac{2(mgh - Fd)}{m}}$

② $K.E_{loss} = P.E_{gain} + \text{Work done against friction}$

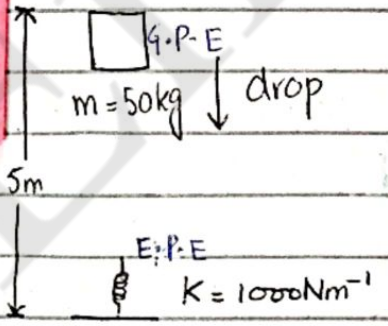
$\frac{1}{2}mv^2 = mgh + Fd$

$v = \sqrt{\frac{2(mgh + Fd)}{m}}$

LOSS = GAIN + W against F

$E_{loss} - E_{gain} = W \text{ against } F$

General eq ↑



max. compression = ?

$G.P.E_{loss} = E.P.E_{gain}$

$mgh = \frac{1}{2}kx^2$

$x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2(5)(10)(5)}{1000}}$

$= \sqrt{5} = 2.2\ m$

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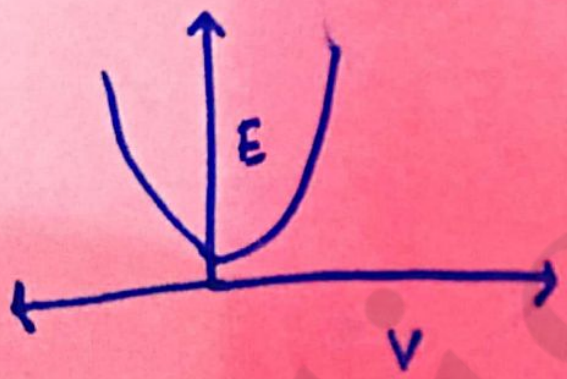
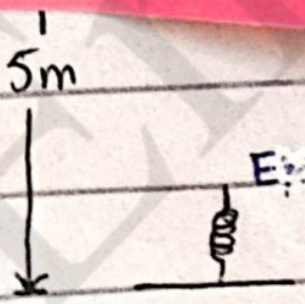
$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

Graph b/n E & V is parabola





max. compression = ?

② K.E = P.E
loss

$$\frac{1}{2}mv^2 = m$$

$$v = \sqrt{\frac{2}{}}$$

LOSS = GAI

$E_{loss} - E_{gain} =$

General e

G.P.E = $mgh = \frac{1}{2}k$
loss

$$x = \sqrt{\frac{2mg}{k}}$$

$= \sqrt{5} =$

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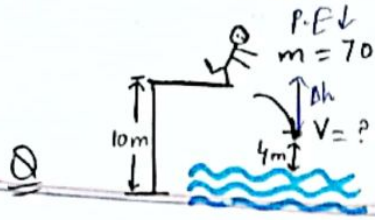
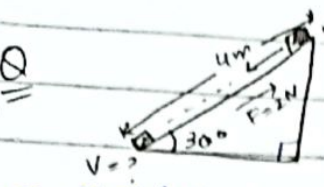


$P.E_{loss} = K.E_{gain}$

$mgh = \frac{1}{2}mv^2$

$v = \sqrt{2g\Delta h} = \sqrt{2(10)(10-4)} = \sqrt{20\times6}$

$= \sqrt{120} \approx 11 m/s$



$P.E_{loss}$ = Gain in K.E + Fd

$mgh = \frac{1}{2}mv^2 + Fd$

$v = \sqrt{2(mgh - Fd)}$

$v = \sqrt{2(\frac{1}{2})(10)(2) - (2)(4)}$

$v = \sqrt{20-8} = \sqrt{16} = 4 m/s$

To find 'h'

$\because \theta = 30°$

$\sin\theta = \frac{P}{H}$

$\sin 30° = \frac{P}{4} \Rightarrow \frac{1}{2_1} \times 4^2 = 2m$

% Loss in Energy $= \frac{\text{Loss}}{T.E_{(i)}} \times 100 = \frac{Fd}{P.E} \times 100$

$= \frac{P.E - K.E}{P.E} \times 100$

How to calculate % increase / decrease in K.E

$\%$ increase in K.E $= \left(2n + \frac{n^2}{100}\right)\%$

$\%$ decrease in K.E $= \left(2n - \frac{n^2}{100}\right)\%$

Q If P is increased 20%, increase in K.E = ?

$\% K.E = \left(2(20) + \frac{(20)^2}{100}\right) = \left(40 + \frac{400}{100}\right) = 44\%$

How to calculate % increase in momentum ?

$\%$ increase in momentum $= \left(\frac{100 + x}{100}\right)^n - 1$

$x = \%$ increase in K.E

$n = 1/2$ $\because$ $K.E \propto P^2$ i.e $P = \sqrt{K.E} = (KE)^{1/2}$

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Potential Energy

↳ P.E is **state function**, i.e depends upon initial and final position.

Gravitational Potential Energy:

"ΔP.E in gravitational field"

"Workdone in moving a body from infinity to a point inside gravitational field, that workdone is stored in the form of G.P.E"

(Also $a = 0$ i.e $F_{net} = 0$ (without accelerating))

$$G.P.E = mgh = (F_g)h$$

(small height) $h < R$ (Radius of earth)

F_g = const

$$U = G.P.E = -\frac{GMm}{r}$$

$h > R$ or $r > R$

r = distance of body from centre of earth

$F_g \neq 0$

Important points:-

- G.P.E is always determined relative to some arbitrary position, which is assigned as **zero P.E** value. This point is called **reference point / zero level** e.g ground / infinity
- **Earth surface** or **point at infinity** are taken as reference points.
- if $g = 0$, $G.P.E = 0$
  - at infinity: $F_g = 0$
  - at centre of Earth: $F_g = 0$

$e^- \rightarrow \quad \leftarrow p^+$; $\leftarrow e^- \quad p^+ \rightarrow$

in same direction: P.E ↓, K.E ↑

opposite direction: P.E ↑, K.E ↓

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Absolute G·P·E (U) → point in Earth G·F → Infinity

"Energy stored in a body to do workdone in Earth's Gravitational Field in displacing the body from a certain point to infinity where $a_g = 0$, $F_g = 0$"

$$W_{p \to \infty} = U$$

$$U = -\frac{GMm}{r} \quad \because r > R$$

-ve sign → workdone against gravitational force

→ F_g → attractive nature

At Earths surface, $r = R$

$$U = -\frac{GMm}{R}$$

Important points

- if r_{inc} i-e $r \uparrow$, then $U \uparrow$

  Although $r \propto \frac{1}{U}$, but due to (-ve) sign $\because -2 \uparrow > -3$

- if $r_{dec} \downarrow$, $U_{dec} \downarrow$
- if $r = \infty$, $U = 0$

Workdone

- $+W$ → U_{dec}
- $-W$ → U_{inc}

Gravitational Potential (V)

"P·E per unit mass at r from center of Earth"

$$V = \frac{U}{m} = \frac{-GmM}{rm} = -\frac{GM}{r}$$

unit

$J kg^{-1}$, m^2/s^2

SI, SI base

$\frac{kg\, m^2 s^{-2}}{kg}$

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Mass of Earth

5.97×10^{24} kg

Radius of Earth

6×10^{6} m

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Escape Velocity:

"Minimum initial velocity to move body out of earth's gravitation field."

→ Diff for every planet -

EXAMPLES

for earth, $V_{esc} = 11\ km/s$

for moon, $V_{esc} = 2.4\ km/s$

for mars, $V_{esc} = 5\ km/s$

$$V_{esc} = \sqrt{2gR}$$

$$V_{esc} = \sqrt{\frac{2GM}{R}}$$

$$V_{esc} = \sqrt{2}\ V_0$$

V_0 = orbital velocity

Note:-

V_{esc} does not depends on mass, shape, size, angle of projection of body.

IMPLICATION OF ENERGY LOSSES IN PRACTICAL DEVICES

$$\text{Mechanical Efficiency} = \eta = \frac{\text{Output work}}{\text{input work}}$$

$$\%\ \eta = \frac{W_O}{W_I} \times 100$$

losses=0, ideal machines ⇒ 100% efficient

$\eta = 1$

losses≠0 actual machines ⇒ $\eta < 1$, $\% < 100\%$

↳ Friction → main cause → heat or thermal energy

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Output Work:-

Output work = Load x distance moved by load

$= F_{out} \times d_{out}$

Input work:-

Input work = Effort x Distance through which effort is applied (Effort distance)

$= F_{inp} \times d_{inp}$

$\% \eta = \frac{\text{output}}{\text{input}} \times 100$

For frictional case

$\% \eta = \frac{\text{Output}}{\text{output + Friction force}} \times 100$

$\% \eta = \frac{F_o \times d_o}{F_i \times d_i} \times 100$

$W_i = W_o + Fd_{(f)}$

Q Input 1000J Energy require → lift 10kg load block → $d_o = 6m$

$mg = 10 \times 10 = 100kg$

$\% \eta = ?$

$\% \eta = \frac{100 \times 6 \times 100}{1000} = 60\%$

Q Pulley system → lift load 500N (F_o), → h = 20m (d_o)

Work done against friction = 2000J, $\% \eta = ?$

$\% \eta = \frac{500 \times 20}{(500 \times 20) + 2000} \times 100 = \frac{16,\cancel{000}^{5}}{\cancel{18000}_{3}} \times \frac{50}{100} = \frac{250}{3} = 8.3$

$= 83\%$

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An engine pumps 100 kg of water through height 10m in 5s. Given that efficiency of engine is 60%, what is power of engine?

(A) 33 kW (B) 0.33 kW (C) 3.3 kW (D) 0.3 kW

Solution

$$P_{out} = \frac{mgh}{t} = \frac{100 \times 10 \times 10}{2} = 2\,kW$$

$$\eta = \frac{P_{out}}{P_{inp}} \times 100 \Rightarrow P_{inp} = \frac{P_{out}}{\eta} \times 100$$

$$= \frac{2^{1}}{60_{3}} \times 100$$

$$= 0.33 \times 10 = 3.3\,kW$$

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POWER

"Rate of doing work" → $P = \frac{W}{t}$

$$P = \frac{\Delta P.E}{t} = \frac{mgh}{h}$$

$$P = \frac{\Delta K.E}{t} = \frac{1}{2}\frac{mv^2}{t}$$

SI unit:-
(Watt)
$W = J\,s^{-1} = Nms^{-1}$
Base unit:- kgm^2s^{-3}

"dot/scalar product of force and velocity"

$$P = \vec{F} \cdot \vec{V} = FV\cos\theta$$

if $\theta = 0°$, $P = FV$

$$P_{avg} = F_{net}V = (F - f)V = (F - \mu_k N)V$$

(F: force applied, f: friction)

$$P = F \cdot \frac{\Delta d}{\Delta t}$$

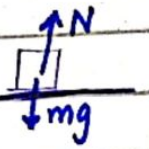
Also,
$P = VI = I^2R = \frac{V^2}{R}$

$P \propto V$

For friction

$f = \mu N$ (μ: coefficient of friction)

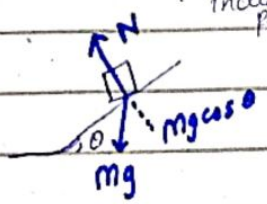
In motion,
Kinetic friction = $f_k = \mu_k N$



for plane

$f_k = \mu_k mg$

For inclined plane



$N = mg\cos\theta$

In angular Motion

"product of torque & ang. velocity"

$P = \tau\omega$

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Otherunits:-

- commercial unit → hp

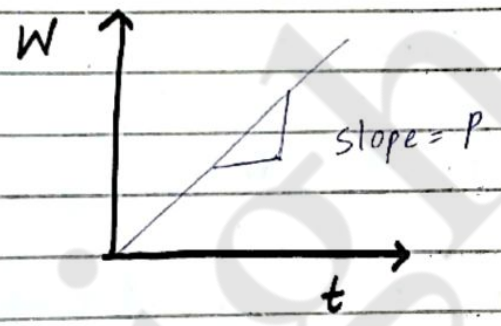
$1hp = 746\ watt = 0.746\ kW$

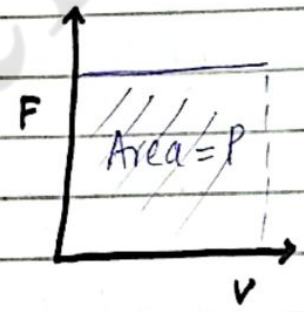
- commercial unit ↑ Killowatt → kW

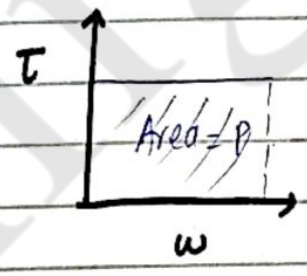
- $A^2\Omega$, VA , $V^2\Omega^{-1}$

I^2R , VI , V^2/R

Graphical Representation:-







Q) Electrical Motor $V = 400V$, $I = 5A$ → lift weight of 2500N upto height of 15m.

$t = ?$

$\because P = \frac{\Delta P.E}{t} \rightarrow VI = \frac{mgh}{t}$

$t = \frac{mgh}{VI} = \frac{\cancel{2500}^{5} \times 15}{\cancel{400} \times \cancel{5}} = \frac{75}{4} \approx 18.7s$

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Energy in KWh

$$\text{Energy (kWh)} = \frac{P(\text{W}) \times t(\text{hr})}{1000}$$

(P in W, t in hr)

1 Kwh = 1 unit

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